Psychological Methods
2007, Vol. 12, No. 1, 45–57

1082-989X/07/$12.00 DOI: 10.1037/1082-989X.12.1.45

Predicting Group-Level Outcome Variables From Variables Measured at the Individual Level: A Latent Variable Multilevel Model

Marcel A. Croon and Marc J. P. M. van Veldhoven
Tilburg University

In multilevel modeling, one often distinguishes between macro-micro and micro-macro situations. In a macro-micro multilevel situation, a dependent variable measured at the lower level is predicted or explained by variables measured at that lower or a higher level. In a micro-macro multilevel situation, a dependent variable defined at the higher group level is predicted or explained on the basis of independent variables measured at the lower individual level. Up until now, multilevel methodology has mainly focused on macro-micro multilevel situations. In this article, a latent variable model is proposed for analyzing data from micro-macro situations. It is shown that regression analyses carried out at the aggregated level result in biased parameter estimates. A method that uses the best linear unbiased predictors of the group means is shown to yield unbiased estimates of the parameters.

Keywords: multilevel analysis, micro-macro multilevel design, best linear unbiased predictor (BLUP)

Supplemental materials: http://dx.doi.org/10.1037/1082-989X.12.1.45.supp

In the last decades, multilevel analysis (Goldstein, 2003; Neuhaus, 1992; Snijders & Bosker, 1999) has gained enormously in popularity in applied and theoretical research in the social and behavioral sciences. As Hofmann (1997) made clear, multilevel models, which describe the interrelationships among variables measured at different levels of observations, are very well-suited for the description and analysis of processes that take place in hierarchically ordered systems. In psychological research, the most often encountered types of hierarchically ordered systems consist of individual persons nested within higher units like cultures, organizations, schools, departments, or teams. A different kind of hierarchically ordered system is obtained when measurement occasions or variables are nested within individual persons, who then constitute the higher level of the hierarchy. In this article, only the former category of multilevel applications in psychology with the individual at the lowest level of analysis will be considered.

By explicitly modeling the relationship between variables at different levels of the hierarchy, multilevel models reduce the risk of succumbing to either the ecological or the atomistic fallacy (Hannan, 1971). When researchers generalize findings from the aggregated to the individual level, they commit the ecological fallacy; when they attempt to generalize from the individual to the aggregate level, however, they fall prey to the atomistic or individualistic fallacy. In both cases, results are generalized to an inappropriate level, because relationships among variables that hold at one level do not necessarily hold at another level of the hierarchy. Multilevel analysis may help in avoiding entrapment in either fallacy by requiring explicit modeling of how variables at different levels affect each other. They may help in understanding cross-level interactions that occur when the effects that explanatory variables at a particular level have on the outcome variable depend on the value of explanatory variables at a different level.

In their discussion of situations in which multilevel modeling may be appropriate, Snijders and Bosker (1999, Chapter 2) distinguished between *macro-micro* and *micro-macro* situations. In macro-micro situations, a dependent variable Y measured at the lower (individual) level is assumed to be

Marcel A. Croon, Department of Statistics and Methodology, Faculty of Social Sciences, Tilburg University, Tilburg, the Netherlands; Marc J. P. M. van Veldhoven, Department of Human Resource Studies, Faculty of Social Sciences, Tilburg University.

We thank the Rabobank Group for providing the opportunity to analyze data from their particular implementation of a balanced scorecard system and publish scientifically about the findings. The data were provided under agreement of confidentiality. We also thank Jacques Hagenaars for his comments on a previous version of the article.

Correspondence concerning this article should be addressed to Marcel A. Croon, Department of Statistics and Methodology, Tilburg University, P.O. Box 90153, 5000 LE Tilburg, the Netherlands. E-mail: m.a.croon@uvt.nl

influenced by explanatory variable *X,* also measured at the lower level, and by explanatory variable *Z,* measured at the higher (group) level. In micro-macro situations, however, a dependent variable *Y* that is measured at the higher (group) level is influenced by explanatory variables that are measured either at the lower (individual) or at the higher (group) level.

Although the basic distinction between these two types of models is well-known, most of the work on multilevel modeling has been confined to situations of the macro-micro type. Snijders and Bosker (1999, p. 12) explicitly stated that they would only discuss the statistical techniques for analyzing multilevel data with the outcome variables measured at the micro level. A similar restriction to models that are appropriate for the macro-micro situation is found in the textbooks by Heck and Thomas (2000), Raudenbush and Bryk (2002), and Goldstein (2003). Moreover, software packages like MLwiN and HLM that were designed for the analysis of multilevel data do not explicitly offer methods for analyzing data from a micro-macro research design.

That the study of multilevel models has mainly focused on macro-micro situations does not mean that micro-macro situations do not commonly occur in psychological research. Many examples exist of situations in which psychological researchers are confronted with the problem of how to predict or explain higher level outcome variables from psychological variables measured at the individual level. These examples can be found in various areas of psychological inquiry, like work and organizational psychology (DeShon, Kozlowski, Schmidt, Milner, & Wiechmann, 2004; Schneider, Hanges, Smith, & Salvaggio, 2003), educational psychology (Raudenbush & Willms, 1995; Rutter & Maughan, 2002), cross-cultural psychology (Lim, Bond, & Bond, 2005; van de Vijver & Leung, 2000), and (experimental) personality and social psychology (Day & Carroll, 2004; Kelly & Loving, 2004). All of these areas of research may profit from the development of appropriate methods for the analysis of data collected in a micro-macro design.

We can illustrate the need for an analytical tool in the micro-macro context using psychological effectiveness research as an example. This area of applied psychology concerns the study of how individual psychological variables exert an influence on the performance of higher level units, such as organizations, schools, or teams. Examples in organizational behavior are provided by studies on the implications of employee job satisfaction and commitment for organizational performance (Harter, Schmidt, & Hayes, 2002; Ostroff, 1992) and on the impact on organizational performance of management practices and work climate as perceived by individual employees (Fulmer, Gerhart, & Scott, 2003; Parker et al., 2003; Schneider et al., 2003). In educational psychology, the issue of school effectiveness and its relationships with attributes of the individual students and peer groups within classes and of teachers is a comparable research issue (Raudenbush & Willms, 1995; Rutter & Maughan, 2002). So is the explanation of the effectiveness of job or project teams by attributes of individual team members and their interactions (DeShon et al., 2004; Waller, Conte, Gibson, & Carpenter, 2001). These effectiveness studies share some common methodological characteristics. First, the independent variables are microlevel psychological attributes. Survey or interview methods are being used to measure individual perceptions of the organization, school, or team or individual attitudes like individual well-being, commitment, et cetera. Second, at the macro level, measures of performance are collected for the organization or school as a whole or for relevant subunits within these organizations or schools, for example, departments, teams, subsidiaries, classes, or locations. It is important to stress here that although some macrolevel outcomes may be derived from aggregated individual responses (e.g., absenteeism rates, mean job satisfaction scores, or percentages of students passing examinations), outcomes in this type of research are often only available at the aggregated level. Common examples are organizational or team productivity figures, sales records, profits/costs ratios, anonymous customer satisfaction ratings, and number of students graduated. As organizations and schools are continuously looking for ways to improve performance, and because research on optimizing the human factor is an integral part of this endeavor, micro-macro designs will continue to appear in this research area.

Classical approaches to cope with situations in which the outcome variable is measured at the group level whereas some of its explanatory variables are measured at the lower individual level consist of either disaggregating or aggregating the data. In the disaggregation approach, individuals receive scores on a group-level variable by assigning them their group score on that variable. All variables are then finally transformed into variables defined at the lower individual level. In the aggregation approach, all variables are transformed into variables measured at the higher group level by assigning each group its average score on any individual-level variable. When disaggregating a group variable, researchers should ensure that all subjects in a particular group receive the same score on the corresponding individual-level variable. Because further statistical analyses are almost always based on models that explicitly postulate variation in the individual scores, they generally yield inaccurate estimates of the standard errors of the model parameters. A similar observation applies when aggregating individual-level variables to the group level: Assigning group means as scores on variables at the group level also reduces the variability in the data, yielding inappropriate estimates of the standard errors of the regression parameters.

Analyses of data measured at different levels should be based on models that explicitly acknowledge the existence of these different levels and that attempt to formulate the interaction between the levels in the production of the outcome variable (Schnake & Dumler, 2003). When the outcome is measured at the individual level, various multilevel methods are available to disentangle the effects of group- and individual-level variables and their cross-level interactions on the outcome variables. Such methods are not yet available when the outcome variable is measured at the group level. In his classification of research situations with reciprocal effects between variables at different levels, Hofmann (2002) proposed no specific method for the case in which a dependent variable with only higher level variance is predicted from independent variables with both lower and higher level variance. His suggestion was to aggregate the independent variables to the group level and to carry out an ordinary least squares (OLS) regression analysis on the aggregated data. One of the few articles that explicitly proposed a multilevel model for data with outcome variables measured at the group level is Griffin (1997), but his approach requires longitudinal data with at least three waves of measurement.

In this article, we propose a multilevel model for the analysis of an outcome variable measured at the group level when some (or all) of its explanatory variables are measured at the individual level. By making explicit the assumptions on which the aggregation approach is based, the model proposed here can be seen as a formalized representation of the aggregation strategy. We show that just substituting group means in a regression analysis for the group outcome variable leads to biased estimates of the true regression coefficients. However, the same analysis also suggests how to adjust the regression analysis to obtain unbiased estimates of the parameters. Before being applied to a real data set, the procedure proposed in this article is first tested in a simulation study. In the final section, we discuss its potentialities and limitations.

A Latent Variable Multilevel Model for Analyzing Aggregated Data

Introducing the Model

We assume that data were collected in G different groups with n_g observations in group g. The number of observations in each group may be constant or varying. The total number of individual observations is $N = \Sigma_g n_g$. We also assume that the scores y_g of groups g on a group-level outcome variable Y may be explained or predicted by one group-level explanatory variable Z with score z_g for group g and one individual-level explanatory variable X with score x_{ig} for individual i in group g. In the next section, the multivariate extension of this model when several explanatory variables are available at both levels is discussed.

Figure 1 graphically represents the model proposed in this article. A latent group-level variable ξ is associated with the explanatory individual-level variable X on which only the groups have score ξ_g. The score x_{ig} for each individual in group g on X is treated as a reflective indicator for that unobserved group score. The unobserved group-level variable ξ may be correlated with the observed group-level variable Z, and both may have an effect on the group-level outcome variable Y. The population means of ξ and Z are denoted by μ_ξ and μ_z, their population variances by σ^2_ξ and σ^2_z, and their covariance by $\sigma_{\xi z}$. Their covariances with the dependent variable Y are $\sigma_{\xi y}$ and σ_{zy}, respectively.

The model shown in Figure 1 defines a set of linear regression equations. The first equation describes the relationship between the group scores on the explanatory variables Z and ξ and the outcome variable Y:

$$y_g = \beta_0 + \beta_1\xi_g + \beta_2 z_g + \epsilon_g. \quad (1)$$

The error or disturbance variable ϵ is assumed to be homoscedastic, that is, to have constant variance σ^2_ϵ for all groups.

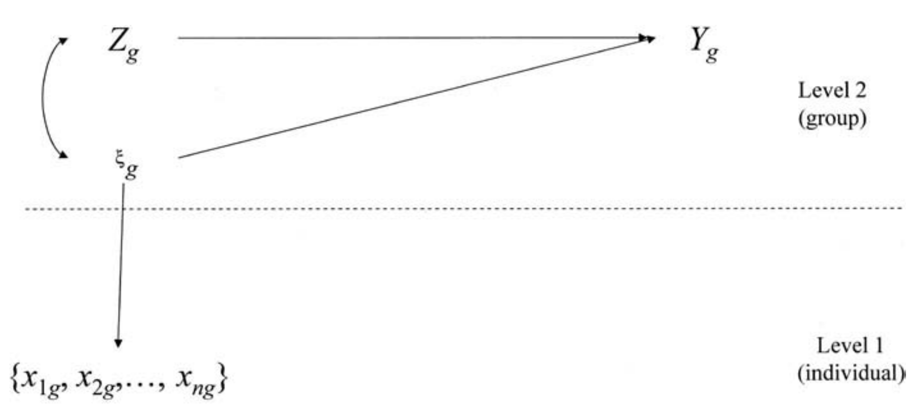

Figure 1. A latent variable multilevel model for outcomes measured at the group level.

The regression parameters in this model satisfy the following relationships:

$$\begin{pmatrix} \sigma_\xi^2 & \sigma_{\xi z} \\ \sigma_{\xi z} & \sigma_z^2 \end{pmatrix} \begin{pmatrix} \beta_1 \\ \beta_2 \end{pmatrix} = \begin{pmatrix} \sigma_{\xi y} \\ \sigma_{zy} \end{pmatrix} \quad (2)$$

and

$$\beta_0 = \mu_y - \beta_1\mu_\xi - \beta_2\mu_z.$$

All three variables are defined at the group level, but because ξ is not directly observed, standard regression analysis cannot be applied to estimate the regression coefficients β_0, β_1, and β_2.

A second set of equations specifies the relationships between the group score ξ_g and the score x_{ig} of each subject in group g:

$$x_{ig} = \xi_g + v_{ig}. \quad (3)$$

The variance of the unobserved latent group score ξ is denoted by σ_ξ^2, the variance of the disturbance term v_{ig}, assumed to be constant for all subjects and groups, by σ_v^2. Moreover, ϵ_g and v_{ig} are assumed to be mutually independent and to be independent from the group variable ξ. The variances σ_ξ^2, σ_ϵ^2, and σ_v^2 are additional parameters in this model. The variance σ_ξ^2 is the between-group variance of X; the variance σ_v^2 is its within-group variance. The intraclass correlation for X is then defined as

$$\rho_X = \frac{\sigma_\xi^2}{\sigma_\xi^2 + \sigma_v^2}.$$

Note that under this model, the lower level units in the same group are considered exchangeable because their scores all have the same relationship to the unobserved group mean ξ_g.

Fitting the Model as a Structural Equation Model

Although the model described above resembles a structural equation model, with Equation 1 as its structural part and Equation 3 as its measurement part, it differs quite fundamentally from the prototypical model of this kind. In a standard structural equation model, the latent variables are measured via indicator variables on which, in principle, all subjects have scores. In the latent variable model proposed here, the subjects themselves act as indicators for the unobserved group score ξ_g. Because each subject belongs to only one group, the indicators for ξ vary among groups, and this fact seems to preclude a straightforward transformation of the model into a standard structural equation model. Recently, however, Bauer (2003), Curran (2003), and Mehta and Neale (2005) have shown how multilevel models can be fitted as structural equation models by treating the lower level units as indicators for latent variables involved in a multilevel model. By treating the macrolevel units as the units of analysis and the microlevel units as indicators, the micro-macro multilevel model can be turned into a structural equation model that can be fitted by any structural equation modeling software package.

To see how this persons-as-variables approach applies in the present case, assume first that group size is constant with $n_g = n$ and that only one explanatory variable X at the individual level is involved in the model. Then n person variables $V_1, V_2, \ldots, V_n$ are defined and assigned scores for each group g by randomly linking each observation within a group with exactly one of the person variables. For instance, if the first observation x_{1g} in group g is linked to V_1, the score of group g on this variable is $v_{g1} = x_{1g}$. Similarly, if the second observation x_{2g} in group g is linked to V_2, the score of group g on this variable is $v_{g2} = x_{2g}$. Continuing in this way, each observation in group g is linked to a unique person variable V_h. The assignment rule by means of which this association is established need not be the same for all groups. The measurement model for the person variables becomes $V_h = \xi + \epsilon_h$, with $1 \leq h \leq n$. To ensure that the relation between the latent variable ξ and each person variable is identical, equality of the variances of all error variables ϵ_h must be imposed. Coupled with the structural model that regresses Y on ξ and Z, the combined model can be tested as a structural equation model. Which particular random assignment of individual observations to person variables is used does not matter for the values of the parameter estimates and their standard errors, but the test statistic and descriptive fit indices vary in value for different random assignments of individuals to person variables. This is a consequence of the fact that the observed variance–covariance matrix **S** for the person variables and group-level explanatory variables depends on the particular assignment used, whereas the fitted variance–covariance matrix $\hat{\boldsymbol{\Sigma}}$ remains the same irrespective of the particular link between individuals and person variables.

If two different individual-level explanatory variables X_1 and X_2 are involved, the previous approach is extended by considering $2n$ person variables V_{1h} and V_{2h}. The first set V_{1h} of n person variables corresponds to the first explanatory variable X_1, and scores on V_{1h} are obtained by randomly distributing the n scores on X_1 over this first set of person variables. The second set V_{2h} of person variables corresponds to the second explanatory variable X_2, and scores on V_{2h} are obtained by randomly distributing the n scores on X_2 over this second set of person variables. Here too the association rules that link individual observations to person variables may differ among groups, but within each group, exactly the same association rule should be used to define the two different sets of person variables. If a particular person i in group g is linked to the person variable V_{1h} for the first explanatory variable X_1, then that same person should also be linked to the hth person variable V_{2h}

for the second explanatory variable X_2. The measurement model for ξ_1 and ξ_2 now becomes $V_{1h} = \xi_1 + \epsilon_{h1}$ and $V_{2h} = \xi_2 + \epsilon_{h2}$. Furthermore, for exchangeability of the individuals within the groups to be represented, additional equality restrictions must be imposed on the variances and covariances of the error terms: $\text{var}(\epsilon_{h1}) = \tau_1^2$, $\text{var}(\epsilon_{h2}) = \tau_2^2$, and $\text{cov}(\epsilon_{h1}, \epsilon_{h2}) = \tau_{12}$, with $1 \le h \le n$ for all hs.

The approach sketched above can easily be extended if more than two explanatory variables at the individual level are involved, but it requires some accommodation when the groups are of unequal size. In this case, the number of person variables to be defined for each individual-level explanatory variable is equal to the maximum of the group sizes. For groups with a smaller number of observations, not all person variables can be assigned scores but will have missing scores. Analyzing the restructured data by means of a structural equation modeling software package that provides full information maximum likelihood estimates with missing data yields the maximum likelihood estimates of the model.

Although theoretically sound, the persons-as-variables approach to fitting multilevel models is only practical for data collected in small groups on a limited number of individual-level explanatory variables, because the number of person variables rapidly increases as a function of maximal group size and the number of individual-level explanatory variables. For the analysis of the real data example discussed in a later section, one would need a prohibitively large number of 1,160 person variables. Moreover, to ensure the exchangeability of observations within groups, one needs to impose a large number of constraints on the error term variances and covariances, which makes the preparations for the actual data analysis tedious and unwieldy. It is clear that the persons-as-variables approach does not make the development of more specific estimation methods for micro-macro models superfluous. The estimation procedure proposed in this article is not a full-information maximum likelihood method but consists of a linear regression analysis carried out on adjusted group means for the individual-level explanatory variables.

Why a Simple Aggregated Analysis Does Not Yield Correct Results

A seemingly appropriate way to obtain good estimates of the regression parameters in Equation 1 would consist of aggregating the individual score x_{ig} to the group level by determining the group mean $\bar{x}_g$ and then regressing the score y_g on both $\bar{x}_g$ and z_g in an OLS multiple regression analysis. However, this aggregated regression analysis will not yield unbiased estimates of the regression parameters. If the score y_g is regressed on $\bar{x}_g$ and z_g, the implicitly used regression model is

$$E(y_g | \bar{x}_g, z_g) = \alpha_0 + \alpha_1 \bar{x}_g + \alpha_2 z_g, \qquad (4)$$

with the parameters α_0, α_1, and α_2 satisfying

$$\begin{pmatrix} \sigma_{\bar{x}}^2 & \sigma_{\bar{x}z} \\ \sigma_{\bar{x}z} & \sigma_z^2 \end{pmatrix} \begin{pmatrix} \alpha_1 \\ \alpha_2 \end{pmatrix} = \begin{pmatrix} \sigma_{\bar{x}y} \\ \sigma_{zy} \end{pmatrix} \qquad (5)$$

and

$$\alpha_0 = \mu_y - \alpha_1 \mu_{\bar{x}} - \alpha_2 \mu_z .$$

The aggregated regression analysis defined by Equation 4 would be appropriate if it would yield values for these unknown parameters α_j that are identical to the corresponding values of β_j from the model described by Equation 1. However, the estimates from the two different regression analyses are generally not equal. The relationship between both sets of parameters can be made explicit in the following way. First, note that Equation 3 implies for the mean of variable X in group g that $\bar{x}_g = \xi_g + \bar{v}_g$, so that for group g consisting of n_g subjects,

$$\sigma_{\bar{x}}^2 = \sigma_\xi^2 + \sigma_v^2 / n_g . \qquad (6)$$

Moreover, because the error term v is uncorrelated with ξ, z, and y, it follows that

$$\sigma_{\bar{x}z} = \sigma_{\xi z} \quad \text{and} \quad \sigma_{\bar{x}y} = \sigma_{\xi y} . \qquad (7)$$

Substitution of Equations 6 and 7 in Equation 5 then yields

$$\begin{pmatrix} \sigma_\xi^2 + \sigma_v^2/n_g & \sigma_{\xi z} \\ \sigma_{\xi z} & \sigma_z^2 \end{pmatrix} \begin{pmatrix} \alpha_1 \\ \alpha_2 \end{pmatrix} = \begin{pmatrix} \sigma_{\xi y} \\ \sigma_{zy} \end{pmatrix} . \qquad (8)$$

Equations 2 and 8 both have the same right part. Setting the two left parts equal to each other and solving for the vector containing α_1 and α_2 yields

$$\begin{pmatrix} \alpha_1 \\ \alpha_2 \end{pmatrix} = \begin{pmatrix} \sigma_\xi^2 + \sigma_v^2/n_g & \sigma_{\xi z} \\ \sigma_{\xi z} & \sigma_z^2 \end{pmatrix}^{-1} \begin{pmatrix} \sigma_\xi^2 & \sigma_{\xi z} \\ \sigma_{\xi z} & \sigma_z^2 \end{pmatrix} \begin{pmatrix} \beta_1 \\ \beta_2 \end{pmatrix} .$$

This may be further simplified to

$$\begin{pmatrix} \alpha_1 \\ \alpha_2 \end{pmatrix} = \begin{pmatrix} w_{g1} & 0 \\ w_{g2} & 1 \end{pmatrix} \begin{pmatrix} \beta_1 \\ \beta_2 \end{pmatrix} ,$$

with

$$w_{g1} = \frac{\sigma_\xi^2 \sigma_z^2 - \sigma_{\xi z}^2}{(\sigma_\xi^2 + \sigma_v^2/n_g)\sigma_z^2 - \sigma_{\xi z}^2}$$

and

$$w_{g2} = \frac{\sigma_{\xi z} \sigma_v^2 / n_g}{(\sigma_\xi^2 + \sigma_v^2/n_g)\sigma_z^2 - \sigma_{\xi z}^2} .$$

The relationship between the two sets of regression parameters may then be written as

$$\alpha_1 = w_{g1}\beta_1 \quad \text{and} \quad \alpha_2 = w_{g2}\beta_1 + \beta_2 . \qquad (9)$$

A similar analysis for the constant coefficient leads to

$$\alpha_0 = \beta_0 + [(1 - w_{g1})\mu_\xi - w_{g2}\mu_z]\beta_1. \qquad (10)$$

These results show that the regression analysis carried out on the aggregated scores will generally yield biased estimates of the true parameters β_0, β_1, and β_2. However, if the term σ_v^2/n_g is equal to zero, we have $w_{g1} = 1$ and $w_{g2} = 0$. In this unrealistic case of no within-group variability on variable *X*, the analysis on the aggregated data will result in an unbiased estimate of the true regression parameters. Note, however, that because $\lim_{n_g\to\infty} w_{g1} = 1$ and $\lim_{n_g\to\infty} w_{g2} = 0$, increasing the group sizes will reduce the bias in the estimates of the regression parameters.

How to Correct the Regression Analysis on the Aggregated Data

Although the results described above point out the deficiency of using aggregated group means as independent variables in an OLS regression analysis, they also provide the rationale for an adjustment of an OLS regression analysis that yields unbiased estimates of the regression parameters. Substitution of Expressions 9 and 10 in Equation 4 and further simplifications yield

$$E(y_g|\bar{x}_g, z_g) = \beta_0 + \beta_1 \times [(1 - w_{g1})\mu_\xi + w_{g1}\bar{x}_g + w_{g2}(z_g - \mu_z)] + \beta_2 z_g.$$

To obtain unbiased estimates of the true regression coefficients β_j, one should not regress the scores y_g on the group means $\bar{x}_g$ and z_g but on adjusted group means $\tilde{x}_g$ and z_g, with

$$\tilde{x}_g = (1 - w_{g1})\mu_\xi + w_{g1}\bar{x}_g + w_{g2}(z_g - \mu_z). \qquad (11)$$

The adjusted group mean $\tilde{x}_g$ has the following very important property:

$$\tilde{x}_g = E(\xi_g|x_{1g}, \ldots, x_{n_g g}, z_g),$$

that is, the adjusted group mean $\tilde{x}_g$ is the expected value of ξ_g, taking all the observed scores on the individual- and group-level explanatory variables in group *g* into account. It is the best linear unbiased predictor (BLUP) of the latent group score ξ_g when individual data on *X* and group data on *Z* are available for each group (Laird & Ware, 1982; Searle, Casella, & McCulloch, 1992). The BLUP $\tilde{\xi}_g$ of the random variable ξ_g is the linear combination of the scores x_{1g}, $x_{2g}, \ldots, x_{n_g g}$ and z_g that provides an unbiased estimate of ξ_g and minimizes $E(\tilde{\xi}_g - \xi_g)^2$. Because of the exchangeability of the individuals within a group, their scores have constant weights in the expression for the BLUP, which implies that the group mean $\bar{x}_g$ is sufficient for the prediction of ξ_g. Regressing ξ_g on $\bar{x}_g$ and z_g in their joint distribution leads to Equation 11.

Regressing the group score *Y* on the adjusted mean $\tilde{X}$ and the explanatory group variable *Z* by means of OLS guarantees unbiased estimates of the regression coefficients. For these estimates also to be efficient, the underlying regression model has to satisfy homoscedasticity. One can prove, however, that

$$\mathrm{var}(y_g|\tilde{x}_g, z_g) = \sigma_\epsilon^2 + \frac{(\sigma_z^2\sigma_{y\xi} - \sigma_{\xi z}^2\sigma_{yz})^2\sigma_v^2/n_g}{(\sigma_\xi^2\sigma_z^2 - \sigma_{\xi z}^2)[(\sigma_\xi^2 + \sigma_v^2/n_g)\sigma_z^2 - \sigma_{\xi z}^2]},$$

which shows that the conditional variance under the regression model is a function of group size n_g. Unless group size n_g is constant, the fitted regression model is heteroscedastic and, consequently, OLS regression of *Y* on $\tilde{X}$ and *Z* yields suboptimal estimates of the standard errors of the regression coefficients. A correction procedure developed by econometricians to obtain unbiased estimates of the standard errors of the regression parameters in case of heteroscedasticity is discussed in a later section.

When there is no explanatory group variable *Z*, its contribution to the adjustment of the group means is removed. Then

$$\tilde{x}_g = (1 - w_g)\mu_\xi + w_g\bar{x}_g, \qquad (12)$$

with the weight w_g given by

$$w_g = \frac{\sigma_\xi^2}{\sigma_\xi^2 + \sigma_v^2/n_g}.$$

The adjusted group mean $\tilde{x}_g$ is now obtained by shrinking the observed group mean $\bar{x}_g$ in the direction of the overall mean μ_ξ. The conditional variance is given by

$$\mathrm{var}(y_g|\tilde{x}_g) = \sigma_\epsilon^2 + \frac{\sigma_{y\xi}^2\sigma_v^2/n_g}{\sigma_\xi^2(\sigma_\xi^2 + \sigma_v^2/n_g)}.$$

Unless group size n_g is constant, the regression model is not homoscedastic.[1]

The General Case With Arbitrary Numbers of Explanatory Variables at Both Levels

The same principle for adjusting the observed group means before entering them in a regression analysis applies if several different explanatory variables at the individual and group levels are available.

Let *p* different individual-level explanatory variables be given. The corresponding *p*-dimensional latent variable $\boldsymbol{\xi}_g$ has mean population vector $\boldsymbol{\mu}_\xi$ and between-group

[1] It is important to note that the adjusted regression analysis should be carried out on the raw data and not on group-centered data. Group centering of the individual-level explanatory variables would remove all of the between-group variability and annihilate their correlations with the group-dependent variable.

covariance matrix $\boldsymbol{\Sigma}_{\xi\xi}$. Let a q-dimensional group-level explanatory variable $\boldsymbol{Z}$ be given with mean vector $\boldsymbol{\mu}_z$ and covariance matrix $\boldsymbol{\Sigma}_{zz}$. Each individual i in group g has an observed score on the individual-level explanatory variables; these scores are inscribed in a p-dimensional observed vector $\mathbf{x}_{ig}$. The group means are denoted by $\bar{\boldsymbol{x}}_g$. The scores on the group-level variable $\boldsymbol{Z}$ are inscribed in the q-dimensional vector z_g. The relationship between the individual vector score $\boldsymbol{x}_{ig}$ and the group latent vector score $\boldsymbol{\xi}_g$ is given by $\boldsymbol{x}_{ig} = \boldsymbol{\xi}_g + \boldsymbol{v}_{ig}$, in which $\boldsymbol{v}_{ig}$ is a p-dimensional vector of disturbance terms with covariance matrix $\boldsymbol{\Sigma}_{vv}$, the within-group covariance matrix for the observed score $\boldsymbol{X}$. It is not assumed that $\boldsymbol{\Sigma}_{vv}$ is diagonal because the error terms for different variables may be correlated. The covariance matrix $\boldsymbol{\Sigma}_{\xi\xi}$ for the latent variable $\boldsymbol{\xi}_g$ is the between-group covariance matrix for $\boldsymbol{X}$.

The regression model for the (univariate) dependent group variable Y in terms of the $(p + q)$ explanatory variables is

$$y_g = \beta_0 + \boldsymbol{\xi}_g'\boldsymbol{\beta}_1 + \mathbf{z}_g'\boldsymbol{\beta}_2 + \epsilon_g.$$

The variance of the error term ϵ_g is denoted by σ_ϵ^2.

To obtain unbiased estimates of the regression parameters, we should not regress the score y_g on the aggregated means $\bar{\boldsymbol{x}}_g$ and z_g but on the adjusted means $\tilde{\boldsymbol{x}}_g$ and z_g. The adjusted mean $\tilde{\boldsymbol{x}}_g$ is obtained as follows. First, define for each group g a $p \times p$ weight matrix $\boldsymbol{W}_{g1}$ and a $q \times p$ weight matrix $\boldsymbol{W}_{g2}$ as follows:

$$\boldsymbol{W}_{g1} = (\boldsymbol{\Sigma}_{\xi\xi} + \boldsymbol{\Sigma}_{vv}/n_g - \boldsymbol{\Sigma}_{\xi z}\boldsymbol{\Sigma}_{\xi\xi}^{-1}\boldsymbol{\Sigma}_{z\xi})^{-1}(\boldsymbol{\Sigma}_{\xi\xi} - \boldsymbol{\Sigma}_{\xi z}\boldsymbol{\Sigma}_{zz}^{-1}\boldsymbol{\Sigma}_{z\xi})$$

and

$$\boldsymbol{W}_{g2} = \boldsymbol{\Sigma}_{zz}^{-1}\boldsymbol{\Sigma}_{z\xi}(\boldsymbol{I}_{p\times p} - \boldsymbol{W}_{g1}).$$

Then the adjusted group mean is given by

$$\tilde{\boldsymbol{x}}_g' = \boldsymbol{\mu}_\xi'(\boldsymbol{I}_{p\times p} - \boldsymbol{W}_{g1}) + \bar{\boldsymbol{x}}_g'\boldsymbol{W}_{g1} + (z_g - \boldsymbol{\mu}_z)'\boldsymbol{W}_{g2}.$$

Regressing Y on the adjusted mean $\tilde{\boldsymbol{X}}$ and the observed group score $\boldsymbol{Z}$ yields unbiased estimates of the true regression coefficients.

When no explanatory group variable $\boldsymbol{Z}$ is available, only the first weight matrix $\boldsymbol{W}_{g1}$ has to be considered. Its expression simplifies to

$$\boldsymbol{W}_{g1} = (\boldsymbol{\Sigma}_{\xi\xi} + \boldsymbol{\Sigma}_{vv}/n_g)^{-1}\boldsymbol{\Sigma}_{\xi\xi}$$

and the adjusted group mean is given by

$$\tilde{\boldsymbol{x}}_g' = \boldsymbol{\mu}_\xi'(\boldsymbol{I}_{p\times p} - \boldsymbol{W}_{g1}) + \bar{\boldsymbol{x}}_g'\boldsymbol{W}_{g1}.$$

OLS regression analysis of Y on $\tilde{\boldsymbol{X}}$ and $\boldsymbol{Z}$ does not result in unbiased estimates of the standard errors of the regression parameters unless the groups are of the same size. If group sizes are unequal, the true regression model is not homoscedastic and OLS regression analysis is not fully efficient. In the next section, we describe how to obtain consistent estimates of the standard errors after carrying out an OLS regression analysis.

A Stepwise Estimation Procedure

The previous results form the basis for a stepwise estimation procedure. To be able to carry out a regression analysis of Y on the adjusted group mean $\tilde{\boldsymbol{X}}$ and the explanatory group variable $\boldsymbol{Z}$, one has to determine the weight matrices $\boldsymbol{W}_{g1}$ and $\boldsymbol{W}_{g2}$, which require estimates of the unknown variance and covariance matrices $\boldsymbol{\Sigma}_{\xi\xi}$, $\boldsymbol{\Sigma}_{vv}$, $\boldsymbol{\Sigma}_{\xi z}$, and $\boldsymbol{\Sigma}_{zz}$ and of the mean vectors $\boldsymbol{\mu}_\xi$ and $\boldsymbol{\mu}_z$. The following stepwise procedure can be used to obtain unbiased estimates of all the parameters.

1. In a first step, all parameters necessary to determine the weight matrices $\boldsymbol{W}_{g1}$ and $\boldsymbol{W}_{g2}$ are estimated. The procedures described below guarantee unbiasedness or at least the consistency of these estimates.
   a. The parameters that pertain to the distribution of $\boldsymbol{Z}$ are estimated by equating them to their corresponding sample values: $\hat{\boldsymbol{\mu}}_z = \bar{z}$ and $\hat{\boldsymbol{\Sigma}}_{zz} = \boldsymbol{S}_{zz}$. The computation of the means, variances, and covariances for $\boldsymbol{Z}$ are carried out at the group level.
   b. The population mean vector $\boldsymbol{\mu}_\xi$ is estimated by the overall mean vector $\bar{\boldsymbol{x}}$:
   $$\hat{\boldsymbol{\mu}}_\xi = \bar{\boldsymbol{x}} = \frac{\sum_g \sum_i \boldsymbol{x}_{ig}}{N}.$$
   c. The covariance matrix $\boldsymbol{\Sigma}_{z\xi}$ is estimated by the matrix $\boldsymbol{S}_{z\bar{x}}$, which contains the covariances between the variable $\boldsymbol{Z}$ and the uncorrected group mean $\bar{\boldsymbol{X}}$. Under the model formulated above, $\boldsymbol{S}_{z\bar{x}}$ is a consistent estimate of $\boldsymbol{\Sigma}_{z\xi}$.
   d. For the between- and within-group covariance matrices $\boldsymbol{\Sigma}_{\xi\xi}$ and $\boldsymbol{\Sigma}_{vv}$, their multivariate analysis of variance estimates are determined under a one-way classification, with the individual-level explanatory variables here as dependent variables. First, define the matrices
   $$\boldsymbol{SSA} = \sum_g n_g(\bar{\boldsymbol{x}}_g - \bar{\boldsymbol{x}})(\bar{\boldsymbol{x}}_g - \bar{\boldsymbol{x}})',$$
   $$\boldsymbol{MSA} = \boldsymbol{SSA}/(N - G),$$
   $$\boldsymbol{SSE} = \sum_i \sum_g (\boldsymbol{x}_{ig} - \bar{\boldsymbol{x}}_g)(\boldsymbol{x}_{ig} - \bar{\boldsymbol{x}}_g)',$$
   and
   $$\boldsymbol{MSE} = \boldsymbol{SSE}/(G - 1).$$

Then we have for the estimates of the between- and within-group covariance matrices

$$\hat{\boldsymbol{\Sigma}}_{vv} = \boldsymbol{MSE}$$

and

$$\hat{\boldsymbol{\Sigma}}_{\xi\xi} = \frac{N(G-1)}{N^2 - \sum_g n_g^2}(\boldsymbol{MSA} - \boldsymbol{MSE}).$$

For a derivation of these results, see Searle et al. (1992, Section 3.6) for the univariate case. Their results are easily generalized to the multivariate case (Muthén, 1994).

2. Determine the adjusted group mean $\tilde{\boldsymbol{x}}_g$. If group size varies across groups, the weight matrices $\boldsymbol{W}_{g1}$ and $\boldsymbol{W}_{g2}$ have to be determined for each group separately. If group size is constant, the same weight matrices apply for all groups.
3. Carry out an OLS regression analysis of Y on $\tilde{\boldsymbol{X}}$ and $\boldsymbol{Z}$.
4. If group size is not constant, the heteroscedasticity-consistent covariance matrix estimator defined by White (1980) and modified by Davidson and MacKinnon (1993, Section 16.3) may be determined. First, define matrix $\boldsymbol{U}$ as $\boldsymbol{U} = (\tilde{\boldsymbol{X}}|\boldsymbol{Z})$ and let h_g be the gth diagonal element of $\boldsymbol{U}(\boldsymbol{U}'\boldsymbol{U})^{-1}\boldsymbol{U}$. With e_g denoting the estimate of the error term for group g on the basis of the OLS regression procedure carried out in the previous step, define the weight d_g as

$$d_g = \frac{e_g^2}{1 - h_g}$$

and let $\boldsymbol{D}$ be the diagonal matrix containing this weight. The heteroscedasticity-consistent covariance matrix estimator $\boldsymbol{V}$ is then given by

$$\boldsymbol{V} = (\boldsymbol{U}'\boldsymbol{U})^{-1}(\boldsymbol{U}'\boldsymbol{D}\boldsymbol{U})(\boldsymbol{U}'\boldsymbol{U})^{-1}.$$

The asymptotically consistent estimates of the standard errors of the regression parameters are the square roots of the diagonal elements of $\boldsymbol{V}$. Whether this correction procedure for countering the effects of heteroscedasticity should be applied routinely remains to be seen. A small simulation study, whose results are not presented here in detail, showed that the standard errors were, on average, only reduced by 1–2% of their uncorrected value, depending on how markedly the group sizes differed. These results seem to indicate that not much efficiency is lost by carrying out an OLS regression analysis on the adjusted group means.

A Simulation Study of Bias Reduction in Parameter Estimates

We carried out a simulation study to examine how the accuracy of the adjusted regression analysis was affected by the number of groups, the number of observations per group, the correlation between the explanatory variables at the group and individual levels, and the intraclass correlation of the individual-level explanatory variable. Data were only generated for the case of one explanatory group variable Z and one individual-level explanatory variable ξ. Both variables were standard, but their correlation was varied at two levels: either $\rho_{z\xi} = .0$ or $\rho_{z\xi} = .3$. A single population regression equation for Y was considered:

$$y_g = 0.3 + 0.3z_g + 0.3\xi_g + \epsilon_g,$$

with the error term ϵ_g normally distributed with a mean equal to zero and variance $\sigma_\epsilon^2 = 0.35$. The regression weights of the explanatory variables were chosen so as to represent moderately large effects and were not systematically manipulated in this simulation study.

The individual observed score x_{ig} was obtained by $x_{ig} = \xi_g + v_{ig}$, with v_{ig} normally distributed with mean 0 and variance equal to either 4 or 9. In this way, the intraclass correlation of X was varied at two levels: $\rho_X = .1$ and $\rho_X = .2$.

The number of groups was also varied at two levels: $n_g = 50$ and $n_g = 100$. For the number of observations per group, three levels were defined. The first two levels kept group size constant at either $n_s = 10$ or $n_s = 40$; for the third level, group sizes were varied by randomly selecting either 10 or 40 observations with equal probabilities. By choosing these values for the number of groups and the number of subjects per group, we reduced the risk of obtaining inadmissible solutions.

For each cell of this $2 \times 2 \times 2 \times 3$ design, 1,000 data sets were drawn from the population described above. For each data set, both an unadjusted regression analysis and an adjusted regression analysis were carried out, and for each of the three regression coefficients, the percentage of bias was determined. In these computations, only admissible solutions were taken into account. A solution was considered admissible if the estimated variance–covariance matrix of $(\boldsymbol{Z}, \boldsymbol{\xi})$ was positive-definite. Some inadmissible solutions occurred, but only in three cells of the design matrix. In the cells with $\rho_{z\xi} = .3$, $\rho_X = .1$, and $n_s = 10$ or randomly drawn from the equal probability mixture, the percentage of inadmissible solutions was 1.6% and 0.5%, respectively; for the cell with $\rho_{z\xi} = .0$, $\rho_X = .1$, and $n_s = 10$, that percentage was 0.7%. In no other cell did inadmissible solutions occur.[2] Table 1 gives the average percentage of bias for all conditions.

A few clear conclusions can be drawn from the results of this small simulation study. First, the estimates of regression coefficient β_2 of the individual-level explanatory variable show a severe downward bias in the unadjusted regression analyses. A linear regression analysis of the mean percentage of bias scores on the dummy variables representing the manipulated factors indicated that two factors had a large impact on the bias: the number of observations per group and the size of the intraclass correlation. Bias for β_2 was smaller for larger groups and for higher values of ρ_X. The effects of the size of the correlation $\rho_{z\xi}$ and the number of groups were much smaller. The bias in the estimation of β_1, the regression coefficient of *Z,* was less extreme and showed a different pattern. The size of the correlation $\rho_{z\xi}$ and the number of observations per group were now the most important determinants of bias. Finally, there was no apparent systematic bias in the unadjusted estimate of the constant coefficient, but a further analysis showed that this was a direct consequence of the assumption that ξ and Z had zero means in the population model. Because the constant coefficient is generally not very relevant in the substantive interpretation of the results, we did not investigate further the factors that affect its bias.

The adjusted regression analysis reduced the bias in the parameter estimates quite well. Averaged over all conditions, the percentage of bias in estimating β_1 was reduced from 4.2% to −0.5%; for β_2, the reduction in bias from −27.1% to 2.3% was even more extreme. For the bias in β_2 remaining after correction, no systematic pattern was found in terms of the manipulated design factors. For the remaining bias in β_1 after correction, both the size of $\rho_{z\xi}$ and the number of observations per group still had an effect.

An Application on Data From Psychological Climate Research

A large part of the scientific research in occupational and organizational psychology focuses on the effects of individual psychological attributes and processes on the performance of organizations (Spector, 2003; Wright & Gardner, 2003). The real data application discussed in this section originates from a study on the relationship between psychological climate and organizational performance (Fulmer, Gerhart, & Scott, 2003; Gelade & Ivery, 2003; Parker et al., 2003; Patterson, Warr, & West, 2004; Schneider et al., 2003).

The data analyzed here were collected in a large financial services organization in the Netherlands during the years 2000–2002 as part of a regular company management survey based on the balanced scorecard (Kaplan & Norton, 1996). Questionnaire data from 14,265 individual employees were obtained in 175 business units. The number of employees per unit differed considerably, running from 17 to 295, with an average number of 81.5 employees per unit.

The group-level outcome variable of interest here is the financial performance of business units, which is operationalized as a yearly business unit productivity index. This index is a simple profits/costs ratio. Profits are operationalized as the gross profits plus the returns on equity. Costs include operational costs and depreciation allowances. The scores on four questionnaire scales from the company survey are taken as explanatory variables at the individual level. The company survey comprises various scales that are widely used in the Netherlands for measuring psychological climate factors (Patterson et al., 2005). The following four scales were selected: Role Overload, Effective Leadership, Innovation, and Cooperation Among Departments. With Cronbach's alphas ranging from .81 to .93, the four scales exhibit good internal consistency at the individual level. The intraclass correlation coefficients for the four scales were .048, .066, .132, and .099, respectively, which were all significantly different from zero (Donner, 1986). Their values indicated that the between-groups differences on the four variables were large enough to include them as explanatory variables for the group-dependent variable. The analysis reported below did not include explanatory variables measured at the group level.

Table 2 summarizes the results of several analyses that were carried out on these data. The upper part of the table gives the results obtained on the full sample of 14,265 subjects in the 175 units. In the left upper part, the results of the unadjusted aggregated regression analysis are reported; in the right upper part, the results of the adjusted analysis can be found. For both analyses, the unstandardized regression coefficients (*b*), their standard errors, their standardized values (β), and the corresponding *t* values are given. For the adjusted regression analysis, an additional column is included that gives the adjusted standard errors for the heteroscedastic case after application of the White–Davidson–MacKinnon (WDM) procedure. The values of the *t* statistic in the adjusted analysis are based on the standard errors from the WDM procedure. Table 2 shows that the WDM-corrected standard errors are only slightly smaller than the uncorrected ones.

As can be expected on the basis of the large group sizes and the large full sample size, the results of both analyses are very similar. Both analyses showed that only Role

[2] Another simulation study not reported here showed that the probability of obtaining an inadmissable solution increased with decreasing intraclass correlation ρ_X, increasing correlation $\rho_{z\xi}$, and decreasing n_s. The effect of the intraclass correlation was especially strong: The lower this coefficient, the smaller the between-groups differences and the more difficult it becomes to relate the explanatory variable to the group-level outcome variable.

Table 1
Mean Percentages of Bias for the Three Regression Coefficients in the Unadjusted (URA) and Adjusted (ARA) Regression Analyses

				β_0		β_1		β_2	
$\rho_{z\xi}$	ρ_X	n_g	n_s	URA	ARA	URA	ARA	URA	ARA
.0	.1	50	10	1.3	−0.7	−0.4	−0.2	−48.0	−8.8
.0	.1	50	40	−0.4	1.1	1.0	0.2	−17.1	0.5
.0	.1	50	Mix	0.1	1.1	−1.0	1.5	−35.3	2.8
.0	.1	100	10	0.3	0.4	−0.1	0.0	−48.0	5.4
.0	.1	100	40	−0.8	0.6	−0.5	0.9	−18.4	2.0
.0	.1	100	Mix	−0.2	0.6	−0.7	−1.2	−35.7	2.4
.0	.2	50	10	−0.2	−0.4	−0.4	−0.4	−28.2	2.3
.0	.2	50	40	−0.4	−0.5	1.5	0.7	−7.4	0.0
.0	.2	50	Mix	0.1	1.5	−1.1	0.8	−20.0	1.4
.0	.2	100	10	−0.3	0.6	1.1	0.6	−28.3	0.6
.0	.2	100	40	0.0	0.5	−0.4	−0.2	−8.8	0.3
.0	.2	100	Mix	−0.2	−0.6	0.6	−0.4	−20.3	−0.3
.3	.1	50	10	0.0	−1.3	14.4	−3.6	−49.5	15.1
.3	.1	50	40	0.5	0.6	6.9	−1.4	−19.4	0.7
.3	.1	50	Mix	−1.0	1.5	11.7	−3.8	−37.4	5.3
.3	.1	100	10	0.6	−1.1	15.3	−1.3	−50.4	5.0
.3	.1	100	40	0.4	0.3	6.2	−0.5	−18.8	1.6
.3	.1	100	Mix	1.0	−0.7	12.0	−1.6	−38.0	4.4
.3	.2	50	10	−2.6	1.1	8.9	−3.2	−30.5	6.0
.3	.2	50	40	0.2	−0.5	1.7	1.2	−8.1	2.2
.3	.2	50	Mix	0.2	0.0	5.6	0.3	−19.6	2.5
.3	.2	100	10	0.2	−0.6	9.4	−0.8	−30.1	2.4
.3	.2	100	40	0.0	−0.6	2.6	1.4	−10.2	−1.5
.3	.2	100	Mix	0.1	−0.5	7.1	−0.7	−21.8	2.1

Note. $\rho_{z\xi}$ = correlation of explanatory group variable *Z* and individual-level explanatory variable ξ; ρ_X = intraclass correlation of *X; n_g* = number of groups; n_s = group size.

Overload was a significant predictor of the group-level dependent variable. The regression coefficient of Cooperation Among Departments was not significant at the two-sided 5% level, but, expecting it to have a positive effect, it would be significant at the one-sided 5% level. Note that although the corrected and uncorrected standard errors are very similar, use of the uncorrected values would have resulted in a nonsignificant result for Cooperation Among Departments. The squared multiple correlation coefficient for the unadjusted analysis was .170, with $F(4, 170) = 8.675$, $p < .001$; the adjusted analysis yielded an R^2 of .175, with $F(4, 170) = 9.042$, $p < .001$.

In the lower part of Table 2, we report in a similar way the results of an unadjusted analysis and an adjusted analysis on a subsample randomly selected from the original full sample. To show that the adjusted and unadjusted analyses may yield different results if the group sizes are smaller, we drew a random sample of 10% of the original number of observations (rounded to the nearest integer, if necessary) for each group, and both analyses were carried out on this smaller data set. There are some striking differences between the unadjusted and adjusted analyses in this subsample. Whereas the unadjusted regression analysis indicates that three of the individual-level explanatory variables are significant predictors of the group outcome variable, the adjusted analysis only points in the direction of one explanatory variable, Role Overload, which was also the only significant predictor in the analysis on the full sample.

A comparison of the parameter estimates from the adjusted and unadjusted regression analyses reveals that the unstandardized regression coefficients and their standard errors from the adjusted analysis are generally larger (in absolute value) than the corresponding coefficients from the unadjusted analysis. This is probably related to the fact that the adjustment procedure induces a transformation of the measurement scale of the individual-level explanatory variables in case no group explanatory variables are involved in the analysis. Equation 12 makes clear that the adjustment procedure causes the group mean $\bar{x}_g$ to shrink toward the overall mean μ_ξ, with the consequence that the adjusted mean $\tilde{x}_g$ has less variance than does the unadjusted mean $\bar{x}_g$. Because the response scale of the outcome variable *Y* is not affected by the adjustment procedure, one might expect larger regression coefficients (and standard errors) when regressing *Y* on $\tilde{X}$ rather than on $\bar{X}$.

Table 2
Results of Regression Analyses (RA) of Financial Performance on Four Psychological Climate Scales

	Unadjusted RA				Adjusted RA				
Scale	*b*	β	*SE*	*t*	*b*	β	*SE*	SE_{WDM}	*t*
				Full sample					
RO	0.76	.24	0.23	3.31**	1.02	.25	0.31	0.31	3.26**
EL	0.01	.01	0.22	0.04	0.03	.01	0.30	0.30	0.11
IN	0.26	.14	0.21	1.27	0.29	.13	0.25	0.23	1.14
CO	0.36	.20	0.19	1.89	0.41	.19	0.26	0.22	1.87
				Random 10% subsample					
RO	0.38	.22	0.13	3.05**	2.91	.40	1.28	1.27	2.28*
EL	−0.22	−.18	0.10	−2.28*	1.01	.22	1.18	1.12	0.90
IN	0.17	.12	0.14	1.24	−0.33	−.13	0.55	0.54	−0.60
CO	0.35	.27	0.13	2.80**	0.66	.25	0.49	0.47	1.38

Note. All *t* tests are based on 170 degrees of freedom. RO = Role Overload; EL = Effective Leadership; IN = Innovation; CO = Cooperation Among Departments; SE_{WDM} = adjusted standard errors for the heteroscedastic case after application of the White–Davidson–MacKinnon procedure.
* $p < .05$. ** $p < .01$.

Discussion

The approach proposed in this article to study the relationship between an outcome variable measured at the higher group level and explanatory variables measured at the lower individual level associates a latent variable with each of the explanatory variables at the lower level and treats the individual scores on these variables as reflective indicators for that latent variable. This model is an explicit formalization of the assumptions that underly an aggregated analysis that takes the group means as predictors or explanatory variables. The strategy of computing a group mean for a particular variable only makes sense if that group mean can be considered to be an estimate of a parameter that characterizes the general level of the individual scores within that group. The true scores of these group parameters then define a latent unobserved variable at the group level. Prerequisites for carrying out an analysis according to this model are that the number of groups is not too small and that the intraclass correlations for the individual-level variables are sufficiently large. But these are also the conditions that should be satisfied when carrying out an unadjusted analysis on the aggregated group means. Such an analysis will yield stable and interpretable results only if the number of research units is sufficiently large—in any case, substantially larger than the number of explanatory variables—and the groups means show sufficient variation that is not entirely due to within-group variation. In these respects, both the adjusted and the unadjusted analyses impose the same constraints on the design of the study, and the recommendations formulated by Klein et al. (2000) for multilevel analyses in hierarchical systems also remain valid for the latent variable approach proposed here.

In this article, we estimated the parameters of the latent variable multilevel model by means of a multiple regression analysis on adjusted group means. Our prediction that this procedure results in unbiased estimates of the true regression parameters was confirmed by a simulation study. The stepwise estimation method proposed in this article is a limited information approach that does not directly maximize the complete likelihood function for the data under the model considered. Although the full information maximum likelihood approach would lead to the asymptotically most efficient estimates of the model parameters, the limited information approach is probably not much less efficient. A more systematic comparison of both approaches is needed here. One should also realize, however, that the attractive properties of maximum likelihood estimation only remain valid if the statistical assumptions on which this approach is based are satisfied. A disadvantage of the maximum likelihood approach is that it requires rather complex optimization procedures that are not yet incorporated into any readily available software package. The limited information approach propagated in this article is less computationally involved. An S-Plus Script file with which all analyses reported here have been carried out is available with the electronic version of this article. For applications with a small number of observations per group and a limited number of individual-level explanatory variables, the persons-as-indicators approach (Mehta & Neale, 2005) may also be a viable alternative.

Micro-macro strategies of multilevel modeling (Snijders & Bosker, 1999) are relatively scarce, as we have noted. The latent multilevel model that has been proposed in this article is only a first step in the direction of developing appropriate methods for analyzing data from a micro-macro study. New critical issues arise for future research. First, efforts should be made to develop a reliable numerical and generally applicable procedure that yields the full information maximum likelihood estimates of the model parameters. Next, more complex causal models for more complex relationships among the group-level variables should be considered.

References

Bauer, D. J. (2003). Estimating multilevel linear models as structural equation models. *Journal of Educational and Behavioral Statistics, 28,* 135–167.

Curran, P. J. (2003). Have multilevel models been structural equation models all along? *Multivariate Behavioral Research, 38,* 529–569.

Davidson, R., & MacKinnon, J. G. (1993). *Estimation and inference in econometrics*. Oxford, England: Oxford University Press.

Day, A. L., & Carroll, S. A. (2004). Using an ability-based measure of emotional intelligence to predict individual performance, group performance, and group citizenship behaviours. *Personality and Individual Differences, 36,* 1443–1458.

DeShon, R. P., Kozlowski, S. W. J., Schmidt, A. M., Milner, K. R., & Wiechmann, D. (2004). A multiple-goal, multilevel model of feedback effects on the regulation of individual and team performance. *Journal of Applied Psychology, 89,* 1035–1056.

Donner, A. (1986). A review of inference procedures for the intraclass correlation coefficient in the one-way random effects model. *International Statistical Review, 54,* 67–82.

Fulmer, I. S., Gerhart, B., & Scott, K. S. (2003). Are the 100 best better? An empirical investigation of the relationship between being a great place to work and firm performance. *Personnel Psychology, 56,* 965–993.

Gelade, G. A., & Ivery, M. (2003). The impact of human resource management and work climate on organizational performance. *Personnel Psychology, 24,* 383–404.

Goldstein, H. (2003). *Multilevel statistical models*. London: Arnold.

Griffin, M. A. (1997). Interaction between individuals and situations: Using HLM procedures to estimate reciprocal relationships. *Journal of Management, 23,* 759–773.

Hannan, M. T. (1971). *Aggregation and disaggregation in sociology*. Lexington, MA: Lexington Books.

Harter, J. K., Schmidt, F. L., & Hayes, T. L. (2002). Business-unit-level relationship between employee satisfaction, employee engagement, and business outcome: A meta-analysis. *Journal of Applied Psychology, 87,* 268–279.

Heck, R. H., & Thomas, S. L. (2000). *An introduction to multilevel modeling techniques*. Mahwah, NJ: Erlbaum.

Hofmann, D. A. (1997). An overview of the logic and rationale of hierarchical linear models. *Journal of Management, 23,* 723–744.

Hofmann, D. A. (2002). Issues in multilevel research: Theory development, measurement, and analysis. In S. G. Rogelberg (Ed.), *Handbook of research methods in industrial and organizational psychology* (pp. 247–274). Malden, MA: Blackwell.

Kaplan, R. S., & Norton, D. P. (1996). *The balanced scorecard: Translating strategy into action*. Boston, MA: Harvard Business School Press.

Kelly, J. R., & Loving, T. J. (2004). Time pressure and group performance: Exploring underlying processes in the attention focus model. *Journal of Experimental Social Psychology, 20,* 185–198.

Klein, K. J., Bliese, P. D., Kozlowski, S. W. J., Dansereau, F., Gavin, M. B., Griffin, M. A., et al. (2000). Multilevel analytical techniques: Commonalities, differences, and continuing questions. In K. J. Klein & S. Kozlowski (Eds.), *Multilevel theory, research, and methods in organizations* (pp. 512–553). San Francisco: Jossey-Bass.

Laird, N. M., & Ware, J. H. (1982). Random-effects models for longitudinal data. *Biometrics, 38,* 963–974.

Lim, F., Bond, M. H., & Bond, M. K. (2005). Linking societal and psychological factors to homicide rates across nations. *Journal of Cross-Cultural Psychology, 36,* 515–536.

Mehta, P. A., & Neale, M. C. (2005). People are variables too: Multilevel structural equations modeling. *Psychological Methods, 10,* 259–284.

Muthén, B. O. (1994). Multilevel covariance structure analysis. *Sociological Methods and Research, 22,* 376–398.

Neuhaus, J. M. (1992). Statistical methods for longitudinal and clustered designs with binary responses. *Statistical Methods in Medical Research, 1,* 249–273.

Ostroff, C. (1992). The relationship between satisfaction, attitudes, and performance: An organizational level analysis. *Journal of Applied Psychology, 77,* 963–974.

Parker, C. P., Baltes, B. B., Young, S. A., Huff, J. W., Altmann, R., Lacost, H. A., et al. (2003). Relationship between psychological climate perceptions and work outcomes: A meta-analytic review. *Journal of Organizational Behavior, 24,* 398–416.

Patterson, M., Warr, P., & West, M. (2004). Organizational climate and company productivity: The role of employee affect and employee level. *Journal of Occupational and Organizational Psychology, 77,* 193–216.

Patterson, M. G., West, M. A., Shackleton, V. J., Dawson, J. F., Lawthorn, R., Maitlis, S., et al. (2005). Validating the organizational climate measure: Links to managerial practices, productivity and innovation. *Journal of Organizational Behavior, 26,* 379–408.

Raudenbush, S. W., & Bryk, A. S. (2002). *Hierarchical linear models: Applications and data analysis*. Thousand Oaks, CA: Sage.

Raudenbush, S. W., & Willms, J. D. (1995). The estimation of school effects. *Journal of Educational and Behavioral Statistics, 20,* 307–335.

Rutter, M., & Maughan, B. (2002). School effectiveness findings 1979–2002. *Journal of School Psychology, 40,* 451–475.

Schnake, M. E., & Dumler, M. P. (2003). Levels of measurement and analysis issues in organizational citizenship behaviour research. *Journal of Occupational and Organizational Psychology, 76,* 283–301.

Schneider, B., Hanges, P. J., Smith, D. B., & Salvaggio, A. N. (2003). Which comes first: Employee attitudes or organizational financial and market performance? *Journal of Applied Psychology, 88,* 836–851.

Searle, S. R., Casella, G., & McCulloch, C. E. (1992). *Variance components*. New York: Wiley.

Snijders, T., & Bosker, R. (1999). *Multilevel analysis: An introduction to basic and advanced multilevel modeling*. Thousand Oaks, CA: Sage.

Spector, P. E. (2003). *Industrial and organizational psychology: Research and practice* (3rd ed.). New York: Wiley.

van de Vijver, F. J. R., & Leung, K. (2000). Methodological issues in psychological research on culture. *Journal of Cross-Cultural Psychology, 31,* 33–51.

Waller, M. J., Conte, J. M., Gibson, C. B., & Carpenter, M. A. (2001). The effect of individual perceptions of deadlines on team performance. *Academy of Management Review, 26,* 586–600.

White, H. (1980). A heteroskedasticity-consistent covariance estimator and a direct test of heteroskedasticity. *Econometrica, 48,* 817–838.

Wright, P. M., & Gardner, T. M. (2003). The human resource–firm performance relationship: Methodological and theoretical challenges. In D. Holman, T. D. Wall, C. W. Clegg, P. Sparrow, & A. Howard (Eds.), *The new workplace: A guide to the human impact of modern working practices* (pp. 311–328). New York: Wiley.

Received August 8, 2005
Revision received July 12, 2006
Accepted July 17, 2006 ■